Semi-markov model for a power plant system comprising of three low pressure boilers and fans

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Abstract- A semi markov model has been developed in the present paper for a power plant system comprising of three low pressure boilers and fans. The present paper deals with the system involving one main boiler, a standby redundant unit of three low pressure and three primary fans: PA fan, FD fan and ID fan. The cold standby redundant system involves three low pressure boilers which operate together in order to keep whole system operating whenever failure occurs in the primary boiler. The ability of doing job of three low pressure boilers is equivalent to that of primary boiler. All low pressure boilers start their operation simultaneously whenever failure occurs in the primary boiler. The facility for repair is available based on FCFS (First-cum-First serve) and there is only one repairman available to do the desired job. The reliability and profit analysis has been done for the present model. Various parameters of system effectiveness for the model have been computed using Semi Markov process and Regenerative point technique. Graphical interpretation has also been done to bring out the analysis better.

Keywords- Standby systems; Semi Markov process; Regenerative point technique.

1. INTRODUCTION

The aim of reliability is to make a system more efficient and operating as long as it can. The reliability of a system is defined as the capacity of a system to perform its function for a specified period of time under stated conditions. The literature of reliability is rich with studies consisting of primary unit with identical standby units [1-7] but it still lacks reliability models based on real life data for systems related to power plant. Singh and Taneja [6] have discussed reliability and cost-benefit analysis of a power plant comprising two gas and one steam turbine with scheduled inspection. Comparatively less work has been done related to boilers of a power plan system. Our study deals to fill this gap. The present study involves one primary boiler, three low pressure cold standby boilers and three fans: PA (Primary Air) fan, FD (Force Draught) fan and ID (Induced Draft) fan. The cold standby unit comprises of three low pressure boilers which operate together in case when failure occurs in the primary boiler. Fans play a vital role for the functioning of complete system, thus their working and non-working effects the functioning of the whole system. There is only one technician available to do the desired job. The system comes at halt when any of the fans come across failure or any of the low pressure boilers fail while failure of primary boiler. Various parameters of system effectiveness have been computed such as MTSF (Mean time to system failure), Availability, Busy period of repairman, Profit etc Semi Markov process and Regenerative point technique for the model. Graphical interpretation has also been done to make the analysis better.

2. MODEL DESCRIPTION

A state transition diagram in fig. 1 shows various transitions of the system. The epochs of entry into states 0, 1, 8, 9 and 10 are regenerative points and thus these are regenerative states. The states 2, 3, 4, 5, 6 and 7 are failed states.
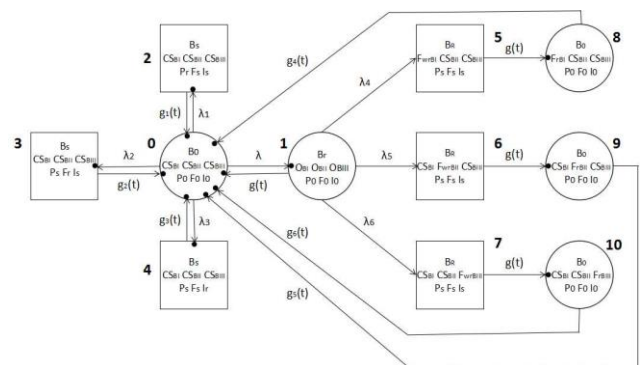

Fig. 1

○ Operating State   ☐ Failed State

3. NOTATIONS

λ	Constant failure rate of main boiler
$\lambda_1 / \lambda_2 / \lambda_3$	Constant failure rate of PA/FD/ID fan
$\lambda_4 / \lambda_5 / \lambda_6$	Constant failure rate of cold standby low pressure boiler (Unit 1/2/3)
$g(t) / G(t)$	pdf/ cdf of repair time for main boiler
$g_1(t) / G_1(t)$	pdf/ cdf of repair time for PA fan

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$g_2(t)/\ G_2(t)$	pdf/ cdf of repair time for FD fan
$g_3(t)/\ G_3(t)$	pdf/ cdf of repair time for ID fan
$g_4(t)/\ G_4(t)$	pdf/ cdf of repair time for low pressure boiler (Unit 1)
$g_5(t)/\ G_5(t)$	pdf/ cdf of repair time for low pressure boiler (Unit 2)
$g_6(t)/\ G_6(t)$	pdf/ cdf of repair time for low pressure boiler (Unit 3)
$B_0/\ P_0/\ F_0/\ I_0$	Main Boiler/ PA/ FD/ ID fan is in operative state
$O_{BI}/\ O_{BII}$	Cold standby low pressure boiler 1/2 is in operative state
$CS_{BI}/\ CS_{BII}/\ CS_{BIII}$	Low pressure boiler 1/2/3 is in cold standby state
$B_r/\ P_r/\ F_r\ /\ I_r$	Main Boiler/ PA/ FD/ ID fan is under repair respectively
$B_S/\ P_S/\ F_S/\ I_S$	Main Boiler/ PA/ FD/ ID fan in switched off state
B_R	Boiler is under repair from the previous state
$F_{rBI}/F_{rBII}/F_{rBIII}$	Cold standby low pressure boiler 1/2/3 is under repair
$F_{wrBI}/F_{wrBII}/F_{wrBIII}$	Cold standby low pressure boiler 1/2/3 is waiting for repair

4. TRANSITION PROBABILITIES:

The non-zero elements p_{ij}, are obtained as under:

$$p_{01} = \frac{\lambda}{\lambda + \lambda_1 + \lambda_2 + \lambda_3}$$

$$p_{02} = \frac{\lambda_1}{\lambda + \lambda_1 + \lambda_2 + \lambda_3}$$

$$p_{03} = \frac{\lambda_2}{\lambda + \lambda_1 + \lambda_2 + \lambda_3}$$

$$p_{04} = \frac{\lambda_3}{\lambda + \lambda_1 + \lambda_2 + \lambda_3}$$

$$p_{10} = g^*(\lambda_4 + \lambda_5 + \lambda_6)$$

$$p_{15} = \frac{\lambda_4[1 - g^*(\lambda_4 + \lambda_5 + \lambda_6)]}{\lambda_4 + \lambda_5 + \lambda_6} = p_{18}^{(5)}$$

$$p_{16} = \frac{\lambda_5[1 - g^*(\lambda_4 + \lambda_5 + \lambda_6)]}{\lambda_4 + \lambda_5 + \lambda_6} = p_{19}^{(6)}$$

$$p_{17} = \frac{\lambda_6[1 - g^*(\lambda_4 + \lambda_5 + \lambda_6)]}{\lambda_4 + \lambda_5 + \lambda_6} = p_{1,10}^{(7)}$$

$$p_{20} = g_1^*(0) \qquad p_{30} = g_2^*(0)$$

$$p_{40} = g_3^*(0) \qquad p_{58} = g^*(0) = p_{69} = p_{7,10}$$

$$p_{80} = g_4^*(0) \qquad p_{90} = g_5^*(0)$$

$$p_{10,0} = g_6^*(0)$$

By these transition probabilities, it can be verified that

$$p_{01} + p_{02} + p_{03} + p_{04} = 1$$

$$p_{10} + p_{15} + p_{16} + p_{17} = 1$$

$$p_{10} + p_{18}^{(5)} + p_{19}^{(6)} + p_{1,10}^{(7)} = 1$$

$$p_{20} = p_{30} = p_{40} = p_{58} = p_{69} = p_{7,10} = p_{80} = p_{90} = p_{10,0} = 1$$

The unconditional mean time taken by the system to transit for any regenerative state j, when it is counted from epoch of entrance into that state i, is mathematically stated as –

$$m_{ij} = \int_0^\infty t\, dQ_{ij}(t) = -q_{ij}^{*\prime}(0), \text{ Thus } -$$

$$m_{01} + m_{02} + m_{03} + m_{04} = \mu_0$$

$$m_{10} + m_{15} + m_{16} + m_{17} = \mu_1$$

$$m_{10} + m_{18}^{(5)} + m_{19}^{(6)} + m_{1,10}^{(7)} = m$$

$$m_{20} = f_1 \qquad m_{30} = f_2$$

$$m_{40} = f_3 \qquad m_{80} = m_1$$

$$m_{90} = m_2 \qquad m_{10,0} = m_3$$

where,

$$m = \int_0^\infty \overline{G}(t)dt \qquad f_1 = \int_0^\infty \overline{G}_1(t)dt$$

$$f_2 = \int_0^\infty \overline{G}_2(t)dt \qquad f_3 = \int_0^\infty \overline{G}_3(t)dt$$

$$m_1 = \int_0^\infty \overline{G}_4(t)dt \qquad m_2 = \int_0^\infty \overline{G}_5(t)dt$$

$$m_3 = \int_0^\infty \overline{G}_6(t)dt$$

The mean sojourn time in the regenerative state i (μ_i) is defined as the time of stay in that state before transition to any other state, then we have:

$$\mu_0 = \frac{1}{\lambda + \lambda_1 + \lambda_2 + \lambda_3} \qquad \mu_1 = \frac{1 - g^*(\lambda_4 + \lambda_5 + \lambda_6)}{\lambda_4 + \lambda_5 + \lambda_6}$$

$$\mu_2 = g_1^*(0) \qquad \mu_3 = g_2^*(0)$$

$$\mu_4 = g_3^*(0) \qquad \mu_5 = g^*(0) = \mu_6 = \mu_7$$

$$\mu_8 = g_4^*(0) \qquad \mu_9 = g_5^*(0)$$

$$\mu_{10} = g_6^*(0)$$

5. MEAN TIME TO SYSTEM FAILURE

The mean time to system failure when the system starts from the state 0, is

$$T_0 = \frac{N}{D}$$

Where

$$N = \mu_0 + \mu_1 p_{01}$$

$$D = 1 - p_{01} p_{10}$$

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6. EXPECTED UPTIME OF THE SYSTEM

The steady state availability of the system is given by:

$$A_0 = \frac{N_1}{D_1}$$

Where

$$N_1 = M_0 + p_{01}[M_1 + M_8 p_{18}^{(5)} + M_9 p_{19}^{(6)} + M_{10} p_{1,10}^{(7)}]$$

$$D_1 = \mu_0 + p_{01}[m + m_1 p_{18}^{(5)} + m_2 p_{19}^{(6)} + m_3 p_{1,10}^{(7)}]$$
$$+ f_1 p_{02} + f_2 p_{03} + f_3 p_{04}$$

7. BUSY PERIOD OF REPAIRMAN

The steady state busy period of the system is given by:

$$B_R = \frac{N_2}{D_1}$$

Where

$$N_2 = p_{01}[W_1 + W_8 p_{18}^{(5)} + W_9 p_{19}^{(6)} + W_{10} p_{1,10}^{(7)}]$$
$$+ W_2 p_{02} + W_3 p_{03} + W_4 p_{04}$$

And D_1 is already specified.

8. EXPECTED NO. OF VISITS OF REPAIRMAN

The steady state expected no. of visits of the repairman is given by:

$$V_R = \frac{N_4}{D_1}$$

Where

$$N_4 = p_{01} + p_{02} + p_{03} + p_{04} = 1$$

And D_1 is already specified.

9. PROFIT ANALYSIS

The expected profit incurred of the system is:

$$P = C_0 A_0 - C_1 B_R - C_2 V_R$$

C_0 = Revenue per unit up time of the system
C_1 = Cost per unit up time for which the repairman is busy in repair
C_2 = Cost per visit of the repairman

10. GRAPHICAL INTERPRETATION AND CONCLUSION

For graphical analysis following particular cases are considered:

$$g(t) = \beta e^{-\beta t} \qquad g_1(t) = p e^{-pt}$$

$$g_2(t) = f e^{-ft} \qquad g_3(t) = i e^{-it}$$

$$g_4(t) = \beta_1 e^{-\beta_1 t} \qquad g_5(t) = \beta_2 e^{-\beta_2 t}$$

$$g_6(t) = \beta_3 e^{-\beta_3 t}$$

$$p_{01} = \frac{\lambda}{\lambda + \lambda_1 + \lambda_2 + \lambda_3} \qquad p_{02} = \frac{\lambda_1}{\lambda + \lambda_1 + \lambda_2 + \lambda_3}$$

$$p_{03} = \frac{\lambda_2}{\lambda + \lambda_1 + \lambda_2 + \lambda_3} \qquad p_{04} = \frac{\lambda_3}{\lambda + \lambda_1 + \lambda_2 + \lambda_3}$$

$$p_{10} = \frac{\beta}{\lambda_4 + \lambda_5 + \lambda_6 + \beta} \qquad p_{15} = \frac{\lambda_4}{\lambda_4 + \lambda_5 + \lambda_6 + \beta} = p_{18}^{(5)}$$

$$p_{16} = \frac{\lambda_5}{\lambda_4 + \lambda_5 + \lambda_6 + \beta} = p_{19}^{(6)} \qquad p_{17} = \frac{\lambda_6}{\lambda_4 + \lambda_5 + \lambda_6 + \beta} = p_{1,10}^{(7)}$$

$$\mu_0 = \frac{1}{\lambda + \lambda_1 + \lambda_2 + \lambda_3} \qquad \mu_1 = \frac{1}{\lambda_4 + \lambda_5 + \lambda_6 + \beta}$$

$$m = \frac{1}{\beta} \qquad f_1 = \frac{1}{p}$$

$$f_2 = \frac{1}{f} \qquad f_3 = \frac{1}{i}$$

$$m_1 = \frac{1}{\beta_1} \qquad m_2 = \frac{1}{\beta_2}$$

$$m_3 = \frac{1}{\beta_3}$$

MTSF V/S RATE OF FAILURE OF MAIN BOILER (λ) FOR DIFFERENT VALUES OF RATE OF FAILURE OF PA FAN (λ₁)

$\lambda_2 = 0.0001788, \lambda_3 = 0.000055, \lambda_4 = 0.0025, \lambda_5 = 0.0050, \lambda_6 = 0.0075,$
$\beta = 0.028047, \beta_1 = 0.00025, \beta_2 = 0.00030, \beta_3 = 0.00035, p = 1.2,$
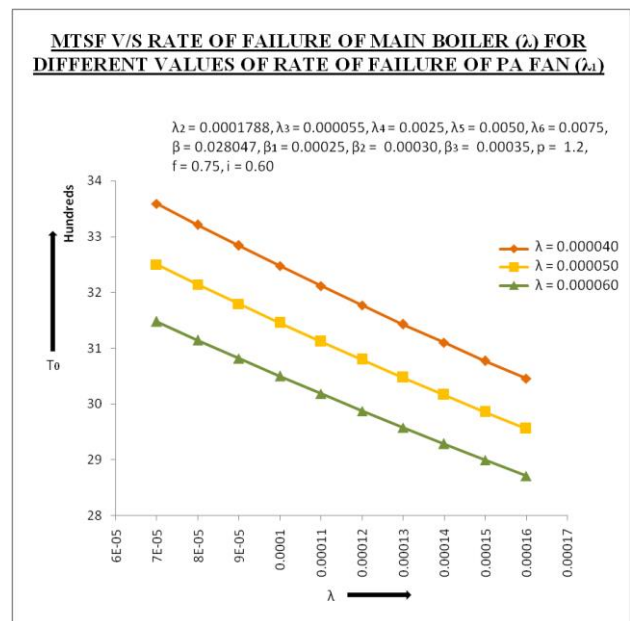$f = 0.75, i = 0.60$

Fig. 2

The behaviour of MTSF w.r.t. failure rate of Main Boiler (λ) for different values of rate of failure of PA fan (λ₁) is shown by Fig. 2. Form the graph, it is easily seen that MTSF decreases while values of the failure rate of main boiler (λ) increases. With the increase in failure rate of PA fan (λ₁), MTSF also decreases.

Fig. 3 reveals the behaviour of profit w.r.t. to failure rate of main boiler (λ) for different values of failure rate of PA fan (λ₁). It is analysed that with the increase in values of failure rate of main boiler (λ), the profit drops. Also, with the increase in failure rate of PA fan (λ₁), the profit declines.
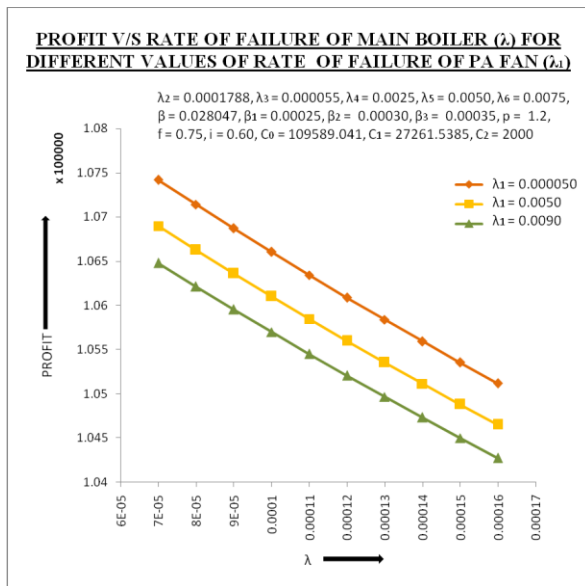
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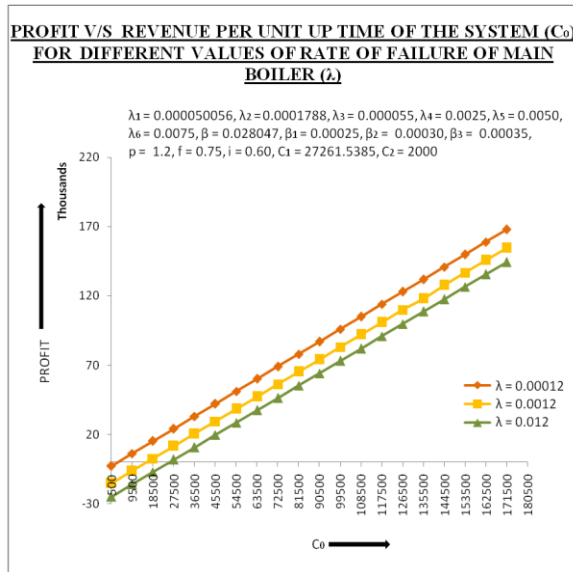*Available online at www.ijrat.org*

Fig. 3

Fig. 4

Fig. 4 shows the behaviour of the profit w.r.t. revenue per unit uptime of the system (C_0) for different values of rate of failure of main boiler (λ). It can be seen from the figure that the profit increases with increase in the values of C_0.

11. CONCLUSION

Following conclusions can be drawn from the graph:

1. For $\alpha = 0.00012$, profit is > or = or < according as C_0 > or = or < 3328, i.e. the revenue per unit uptime of the system in such a way so as to give C_0 not less than 3328 to get positive profit.

2. For $\alpha = 0.0012$, profit is > or = or < according as C_0 > or = or < 15931, i.e. the revenue per unit uptime of the system in such a way so as to give C_0 not less than 15931 to get positive profit.

3. For $\alpha = 0.012$, profit is > or = or < according as C_0 > or = or < 25709, i.e., i.e. the revenue per unit uptime of the system in such a way so as to give C_0 not less than 25709 to get positive profit.

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